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An irreducible representation of $N = 4$ SUSY with two central charges

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Abstract. Using a method of dimensional reduction from an on-shell multiplet in ten space-time dimensions we give the supersymmetry transformation laws of a general irreducible representation of $N = 4$ SUSY with central charges present. With more than one central charge an extra condition is used to give a finite number of components and we give the field transformations and Lagrangian for the case of two central charges.

1. Introduction

One of the basic problems facing construction of extended supergravities and $N = 4$ super Yang-Mills theory is the existence of the $N = 3$ barrier (Rivelles and Taylor 1981, 1983, Taylor 1982a), where N denotes the number of supersymmetry (SUSY) generators in the related global extended SUSY algebra. Out of the the possible three ways to penetrate this barrier (Taylor 1983) only one, that of central charges, seems most likely to allow construction of off-shell extended SUSY theories in terms of fully extended superfields. However, it is necessary that these central charges are degenerate, at least on some multiplets, this degeneracy (Sohnius 1978, Taylor 1980) corresponding to masslessness in higher dimensions. In this case it is well known that the maximal spin in such representations is half that for massive representations, and this spin reduction is precisely that required to avoid the $N = 3$ barrier.

Detailed analysis of $N = 4$ supergravity (Taylor 1982b) shows that SUSY representations are required which have at least two central charges. The construction of irreps of $N = 4$ SUSY with one spin-reducing central charge has been achieved using dimensional reduction by Legendre transformation (Sohnius *et al* 1981). The same method does not seem to work for more than one central charge, since further dimensional reduction beyond one step removes the extra fields introduced at the first step. In order to proceed in the construction of $N = 4$ supergravity (and the situation for supergravities of higher N is expected to be similar) requires the development of irreps of N-SUSY with at least two central charges without using Legendre transformation techniques.

We present here such irreps, obtained directly by techniques using explicitly the on-shell component field transformation laws in ten dimensions. We introduce the further components corresponding to those arising from central charge transformations as derivatives of the usual physical fields, but along the direction of the higher dimensions. These derivatives are evaluated at an arbitrary point (which we can take to be zero), as corresponds to the general theory of integration over central charge dimensions presented elsewhere (Restuccia and Taylor 1983a, Gorse *et al* 1983). In this approach we have shown that space–time $\boldsymbol{R}^4$ is to be regarded as the vertex of a cone Γ in a higher-dimensional space–time involving $\boldsymbol{R}^4$ and the central charge dimensions. Actions may be constructed from which the field equations may be derived in terms of the fields and their derivatives along the central charge dimensions, evaluated at the vertex of Γ.

We proceed by considering the ten-dimensional theory (Gliozzi *et al* 1977, Brink *et al* 1977, Scherk 1979) and the general method of reduction to four dimensions. In § 3 we obtain the transformation laws for the case of one, two and more than two central charges respectively. In § 4 we describe these transformation laws in superfield form and conclude with a discussion of the still unanswered questions raised by our results.

2. Reduction from ten dimensions

In ten dimensions we can take a purely imaginary Majorana representation of the Clifford algebra (Scherk 1979):

$$\{\Gamma^M, \Gamma^N\} = 2\eta^{MN} \tag{2.1}$$

where $\eta^{00} = 1$, $\eta^{II} = -1$, $I = 1, \ldots 9$.

We introduce six real antisymmetric 4×4 matrices satisfying the $\mathrm{SU}(2)\times\mathrm{SU}(2)$ algebra:

$$\begin{aligned}
&\{\alpha^i, \alpha^j\} = \{\beta^i, \beta^j\} = -2\delta^{ij} \qquad i = 1, 2, 3\\
&[\alpha^i, \beta^j] = 0\\
&[\alpha^i, \alpha^j] = -2\varepsilon^{ijk}\alpha^k \qquad [\beta^i, \beta^j] = -2\varepsilon^{ijk}\beta^k.
\end{aligned} \tag{2.2}$$

We can then take our representation to be

$$\begin{aligned}
&\Gamma^\mu = \gamma^\mu \otimes \begin{pmatrix} I_4 & 0 \\ 0 & -I_4 \end{pmatrix} \qquad \Gamma^{3+i} = \mathrm{i}1 \otimes \beta^3 \begin{pmatrix} 0 & \alpha^i \\ \alpha^i & 0 \end{pmatrix}\\
&\Gamma^{6+j} = \mathrm{i}\gamma^5 \otimes \begin{pmatrix} \beta^j & 0 \\ 0 & \beta^3\beta^j\beta^3 \end{pmatrix} \qquad \Gamma^{11} = \Gamma^0 \ldots \Gamma^9 = 1 \otimes \begin{pmatrix} 0 & \beta^3 \\ -\beta^3 & 0 \end{pmatrix}.
\end{aligned} \tag{2.3}$$

This gives, for a Majorana-Weyl spinor in ten dimensions,

$$\psi = \begin{pmatrix} \psi_k \\ (-\beta^3\psi)_k \end{pmatrix} \tag{2.4}$$

where $k = 1, 2, 3, 4$ and ψ_k are four Majorana (real) spinors (i.e. $\psi = \psi^*$ and $\Gamma^{11}\psi = \psi$). We can relabel the ten-vector A_M by

$$A_M = (A_\mu, B_i, B'_j) \qquad i, j = 1, 2, 3 \tag{2.5}$$

and the ten-derivative D_M by

$$D_M = (\partial_\mu, \partial_i, \partial'_j) \tag{2.6}$$

where we take ∂_i, ∂'_j to be the (central charge) derivatives in the six extra dimensions.

If we now take an on-shell representation in ten dimensions we find, on reducing to four, that all boson fields satisfy

$$\Box_{10}\phi = (\Box - \partial_i^2 - \partial_j'^2)\phi = 0 \tag{2.7}$$

and all fermion fields satisfy the spin-reducing equation

$$\not{D}\lambda = 0 \qquad \text{or} \qquad (\not{\partial} I_4 + \mathrm{i}\alpha^i\partial_i + \mathrm{i}\gamma^5\beta^j\partial_j')_{kl}\lambda_l = 0. \tag{2.8}$$

Reducing the ten-dimensional supersymmetry transformation laws will give us, in general, the transformation laws for a four-dimensional representation with central charges present. In § 3 we show how this works in the simplest case.

3. Central charge multiplets

3.1. The reduced multiplet

The simplest on-shell representation in 10 dimensions is well known (Gliozzi *et al* 1977, Brink *et al* 1977, Scherk 1979) and is given by a purely transverse real vector boson and a Majorana–Weyl spinor with supersymmetry transformations

$$\delta A_M = \mathrm{i}\bar{\varepsilon}\Gamma_M\lambda \qquad \delta\lambda = \not{D}\not{A}\varepsilon \tag{3.1}$$

with A_M satisfying

$$D^M A_M = 0. \tag{3.2}$$

By using the blueprint for reduction given we get the four-dimensional versions of (3.1) and (3.2):

$$\begin{gathered}\delta A_\mu = 2\mathrm{i}\bar{\varepsilon}\gamma_\mu\lambda \qquad \delta B_i = 2\bar{\varepsilon}\alpha^i\lambda \qquad \delta B_j' = 2\bar{\varepsilon}\gamma^5\beta^j\lambda \\ \delta\lambda_k = ((\not{\partial} - \mathrm{i}\alpha^i\partial_i + \mathrm{i}\gamma^5\beta^j\partial_j')(\not{A} + \mathrm{i}\alpha^{i'}B_{i'} + \mathrm{i}\gamma^5\beta^{j'}B_{j'}')\varepsilon)_k\end{gathered} \tag{3.3}$$

with A_μ unconstrained and satisfying

$$\partial^\mu A_\mu - \partial_i B_i - \partial_j' B_j' = 0. \tag{3.4}$$

We also have central charge derivatives of these fields which we can treat as independent and we can give the supersymmetry transformations for all these by suitable differentiation of equations (3.3) and using equations (2.7) and (2.8).

For just one non-vanishing central charge it is clear that this leads to a simple doubling of Bose states since $\partial^2 = \Box$ from (2.7) and we get the usual single central charge multiplet. For more than one central charge non-vanishing, equations (2.7) and (2.8) are not strong enough to prevent an infinite number of states and we need to apply an extra condition in this case.

3.2. Two central charges

If we take, as non-vanishing, the derivatives along α^1 and α^2 we get, by using (2.7), as independent fields, for example:

$$A_\mu, \partial_1 A_\mu, \partial_2 A_\mu, \frac{\partial_1^2 A_\mu}{\Box}, \frac{\partial_1\partial_2 A_\mu}{\Box}, \ldots \frac{\partial_1^n A_\mu}{\Box^{[n/2]}}, \frac{\partial_1^{n-1}\partial_2 A_\mu}{\Box^{[n/2]}} \ldots \text{etc.}$$

Clearly there is an infinite number of fields, of both physical and auxiliary dimensions.

We can constrain the multiplet to give a finite number of components by taking the additional conditions

$$\partial_1^2 = a_1^2\Box \qquad \partial_2^2 = a_2^2\Box \tag{3.5}$$

on fields in the multiplet. Then for equation (2.7) to be satisfied we have

$$a_1^2 + a_2^2 = 1 \tag{3.6}$$

and we get as independent Bose fields A_μ, $\partial_1 A_\mu$, $\partial_2 A_\mu$ and $\partial_1\partial_2 A_\mu/\Box$ only so each central charge effectively doubles the number of components.

Writing

$$\phi_{km} = \tfrac{1}{2}(\alpha^3 B_3 + \mathrm{i}\boldsymbol{\beta}\cdot\boldsymbol{B}')_{km} \tag{3.7}$$

so $\phi^*_{km} = (\alpha^1\phi\alpha^1)_{km}$ and $\alpha^1_{km}\phi_{km} = \alpha^2_{km}\phi_{km} = 0$, i.e. $\phi_{km} = \phi_{kmTT}$, we get, using (3.4),

$$\delta\lambda_k = (\partial\!\!\!/ - \mathrm{i}\alpha^1\partial_1 - \mathrm{i}\alpha^2\partial_2)(V\!\!\!/ + \partial\!\!\!/\alpha^3 H/\Box + 2\mathrm{i}\phi_{TT})\varepsilon_k \tag{3.8}$$

where

$$\begin{aligned} &\phi_{TT}\varepsilon_k = \{\phi_{km}\varepsilon_-^m, \phi^*_{km}\varepsilon_{+m}\} \qquad V_\mu = A_{\mu'} = A_\mu - (\partial_\mu/\Box)(\partial_1 B_1 + \partial_2 B_2) \\ &H = -\partial_1 B_2 + \partial_2 B_1 . \end{aligned} \tag{3.9}$$

This gives

$$\begin{aligned} &\delta V_\mu = 2\mathrm{i}\bar\varepsilon\gamma_\mu\lambda \qquad \delta H = 2\mathrm{i}\bar\varepsilon\partial\!\!\!/\alpha^3\lambda \\ &\delta\phi_{km} = \alpha^3_{km}\bar\varepsilon\alpha^3\lambda + \boldsymbol{\beta}_{km}\bar\varepsilon\mathrm{i}\gamma^5\boldsymbol{\beta}\lambda = 2\bar\varepsilon_{+[k}\lambda_{+m]TT} - 2(\alpha^1\bar\varepsilon_-)_{[k}(\alpha^1\lambda)_{-m]TT}. \end{aligned} \tag{3.10}$$

If we now take

$$\partial_1\lambda_k = \mathrm{i}a_1\partial\!\!\!/\alpha^1\chi_{1k} \qquad \partial_2\lambda_k = \mathrm{i}a_2\partial\!\!\!/\alpha^2\chi_{2k} \tag{3.11}$$

we get

$$\lambda_k = -a_1\chi_{1k} - a_2\chi_{2k} \tag{3.12}$$

and

$$\partial_1\partial_2\lambda_k/\Box = a_1 a_2\alpha^3(a_2\chi_1 - a_1\chi_2)_k . \tag{3.13}$$

If we put $V_{1\mu} = V_\mu$, $a_1 A_{1\mu} = \partial_1 V_\mu$, $a_2 A_{2\mu} = \partial_2 V_\mu$ and $a_1 a_2 V_{2\mu} = \partial_1\partial_2 V_\mu/\Box$ and similar for H_1, $\Box\phi_1$, $\Box\phi_2$, H_2, ϕ_{1km}, H_{1km}, H_{2km} and ϕ_{2km} we get the full set of supersymmetry transformations:

$$\begin{aligned} &\delta V_{1\mu} = -2\mathrm{i}\bar\varepsilon\gamma_{\mu'}(a_1\chi_1 + a_2\chi_2) \qquad \delta A_{1\mu} = -2\bar\varepsilon\gamma_{\mu'}\partial\!\!\!/\alpha^1\chi_1 \\ &\delta A_{2\mu} = -2\bar\varepsilon\gamma_{\mu'}\partial\!\!\!/\alpha^2\chi_2 \qquad \delta V_{2\mu} = 2\mathrm{i}\bar\varepsilon\gamma_{\mu'}\alpha^3(a_2\chi_1 - a_1\chi_2) \\ &\delta H_1 = -2\mathrm{i}\bar\varepsilon\partial\!\!\!/\alpha^3(a_1\chi_1 + a_2\chi_2) \qquad \delta\phi_1 = -2\bar\varepsilon\alpha^3\alpha^1\chi_1 \\ &\delta\phi_2 = -2\bar\varepsilon\alpha^3\alpha^2\chi_2 \qquad \delta H_2 = -2\mathrm{i}\bar\varepsilon\partial\!\!\!/(a_2\chi_1 - a_1\chi_2) \\ &\delta\phi_{1km} = -2\bar\varepsilon_{+[k}(a_1\chi_1 + a_2\chi_2)_{m]TT} + 2(\alpha^1\bar\varepsilon_-)_{[k}(a_1\alpha^1\chi_1 + a_2\alpha^1\chi_2)_{m]TT} \\ &\delta H_{1km} = 2\mathrm{i}\bar\varepsilon_{+[k}(\partial\!\!\!/\alpha^1\chi_1)_{m]TT} + 2\mathrm{i}(\alpha^1\bar\varepsilon_-)_{[m}\partial\!\!\!/\chi_{1+m]TT} \\ &\delta H_{2km} = 2\mathrm{i}\bar\varepsilon_{+[k}(\partial\!\!\!/\alpha^2\chi_2)_{m]TT} + 2\mathrm{i}(\alpha^2\bar\varepsilon_-)_{[k}\partial\!\!\!/\chi_{2m]TT} \\ &\delta\phi_{2km} = 2\bar\varepsilon_{+[k}\alpha^3(a_2\chi_1 - a_1\chi_2)_{m]TT} - 2(\alpha^1\bar\varepsilon_-)_{[k}(a_2\alpha^2\chi_1 - a_1\alpha^2\chi_2)_{m]TT} \end{aligned} \tag{3.14}$$

$$
\begin{aligned}
\delta\chi_{1+k} &= \mathrm{i}\alpha^{1}_{km}\not{A}_1\varepsilon^{m}_{-} - a_1\not{\partial}\not{V}_1\varepsilon_{+k} - a_2\alpha^{3}_{km}\not{\partial}\not{V}_2\varepsilon_{+m} + \mathrm{i}\alpha^{1}_{km}\not{\partial}\alpha^{3}_{mn}\phi_1\varepsilon^{n}_{-n} - a_1\alpha^{3}_{km}H_1\varepsilon_{+m} \\
&\quad + a_2H_2\varepsilon_{+k} - 2\alpha^{1}_{km}H^{*}_{1mn}\varepsilon_{+n} - 2\mathrm{i}a_1\not{\partial}\phi_{1km}\varepsilon^{m}_{-} - 2\mathrm{i}a_2\alpha^{3}_{km}\not{\partial}\phi_{2mn}\varepsilon^{n}_{-} \\
\delta\chi_{2+k} &= \mathrm{i}\alpha^{2}_{km}\not{A}_2\varepsilon^{m}_{-} - a_2\not{\partial}\not{V}_1\varepsilon_{+k} + a_1\alpha^{3}_{km}\not{\partial}\not{V}_2\varepsilon_{+m} + \mathrm{i}\alpha^{2}_{km}\not{\partial}\alpha^{3}_{mm}\phi_2\varepsilon^{n} - a_2\alpha^{3}_{km}H_1\varepsilon_{+m} \\
&\quad - a_1H_2\varepsilon_{+k} - 2\alpha^{2}_{km}H^{*}_{2mn}\varepsilon_{+n} - 2\mathrm{i}a_2\not{\partial}\phi_{1km}\varepsilon^{m}_{-} + 2\mathrm{i}a_1\alpha^{3}_{km}\not{\partial}\phi_{2mn}\varepsilon^{n}_{-}
\end{aligned}
$$

with invariant Lagrangian

$$
\begin{aligned}
L &= \tfrac{1}{2}(V_{1\mu}\Box V^{\mu}_{1} + V_{2\mu}\Box V^{\mu}_{2} - A_{1\mu}A^{1\mu} - A_{2\mu}A^{2\mu}) - \tfrac{1}{2}(\phi_1\Box\phi_1 + \phi_2\Box\phi_2 - H_1^2 - H_2^2) \\
&\quad - \tfrac{1}{2}(\phi_{1km}\Box\phi^{*}_{1km} + \phi_{2km}\Box\phi^{*}_{2km} - H_{1km}H^{*}_{1km} - H_{2km}H^{*}_{2km}) \\
&\quad + \mathrm{i}\bar{\chi}_1\not{\partial}\chi_1 + \mathrm{i}\bar{\chi}_2\not{\partial}\chi_2 .
\end{aligned} \tag{3.15}
$$

The commutator of two supersymmetry transformations can then be shown to be

$$[\delta_1, \delta_2] = 4\mathrm{i}\bar{\varepsilon}_1\not{\partial}\varepsilon_2 - 4(\bar{\varepsilon}_1\alpha^{1}\varepsilon_2\partial_1 + \bar{\varepsilon}_1\alpha^{2}\varepsilon_2\partial_2) \tag{3.16}$$

i.e. a space–time translation and two central charge transformations.

The Lagrangian (3.15) is also invariant under the central charge transformations

$$\delta_u = \omega_1\partial_1 + \omega_2\partial_2 \tag{3.17}$$

which gives, for example,

$$
\begin{aligned}
&\delta_u V_{1\mu} = a_1\omega_1A_{1\mu} + a_2\omega_2A_{2\mu} \qquad \delta_uA_{1\mu} = a_1\omega_1\Box V_{1\mu} + a_2\omega_2\Box V_{2\mu} \\
&\delta_uA_{2\mu} = a_1\omega_1\Box V_{2\mu} + a_2\omega_2\Box V_{1\mu} \qquad \delta_u V_{2\mu} = a_1\omega_1A_{2\mu} + a_2\omega_2A_{1\mu} \\
&\delta_u\chi_{1k} = -\mathrm{i}a_1\omega_1\alpha^{1}\not{\partial}(a_1\chi_1 + a_2\chi_2)_k + \mathrm{i}a_2\omega_2\alpha^{2}\not{\partial}(a_2\chi_1 - a_1\chi_2)_k \\
&\delta_u\chi_{2k} = -\mathrm{i}a_1\omega_1\alpha^{1}\not{\partial}(a_2\chi_1 - a_1\chi_2)_k - \mathrm{i}a_2\omega_2\alpha^{2}\not{\partial}(a_1\chi_1 + a_2\chi_2)_k .
\end{aligned} \tag{3.18}
$$

It is clear from (3.14) and (3.18) that, except when $a_1 = 0$ or $a_2 = 0$, the equations cannot be decoupled and we do indeed have a two central charge multiplet. When $a_1 = 0$ or $a_2 = 0$ we get the single central charge case as expected.

3.3. More than two central charges

This extra condition (3.5) generalises easily. For a finite number of components we take

$$\partial_i^2 = a_i^2\Box \qquad \partial_j'^2 = a_j'^2\Box \tag{3.19}$$

with

$$\sum a_i^2 + \sum a_j'^2 = 1. \tag{3.20}$$

We then proceed as before, using (3.4) to rewrite (3.3) with a constrained vector, a traceless scalar tensor and several scalar singlets as in (3.8).

If we consider one complex central charge along say α^1 we get

$$(\not{\partial} + \mathrm{i}\alpha^{1}\partial)\lambda_k = 0 = (\not{\partial} + \mathrm{i}\alpha^{1}\partial^{*})\lambda_k$$

and so $\partial\lambda_k = \partial^{*}\lambda_k$, i.e. ∂ real.

Hence we have to be more careful when we complexify (2.8). This can be done by using the chiral notation taking

$$\mathrm{i}\not{\partial}_{\alpha-}{}^{\alpha+}\lambda_{\alpha+k} = \alpha^{1}_{kl}\partial\lambda^{l}_{\alpha-} \tag{3.21}$$

and taking the complex conjugate of this to define ∂_1^{*}.

For a finite number of components we then need to apply the extra condition

$$\partial^2 = b\Box \tag{3.22}$$

but since we have from (3.21) that

$$\partial\partial^* = \Box \tag{3.23}$$

we get

$$\partial^* = b\partial \tag{3.24}$$

and so $b = e^{-2i\theta}$ for some real θ. In other words ∂ and ∂^* are, in a sense, parallel (this is discussed more fully in Restuccia and Taylor (1983b)).

We can then write

$$\partial = e^{i\theta}\partial_1 \tag{3.25}$$

with ∂_1 real. This gives $\partial_1^2 = \Box$ and we can rewrite (3.21) as

$$i\not{\partial}\lambda_k = \alpha^1(\cos\theta + \gamma_5\sin\theta)\partial_1\lambda \tag{3.26}$$

and we have for (3.3)

$$\delta\lambda_k = (\not{\partial} - i\alpha^1\partial_1(\cos\theta - \sin\theta\,\gamma_5))(\not{A} + i\alpha^{i'}B_{i'} + i\gamma^5\beta^{j'}B'_{j'})\varepsilon_k \tag{3.27}$$

A second complex central charge along α^2 would give us

$$i\not{\partial}\lambda_k = (\cos\theta + \gamma_5\sin\theta)(\alpha^1\partial_1 + \alpha^2\partial_2)\lambda \tag{3.28}$$

which, squared, gives $\partial_1^2 + \partial_2^2 = \Box$. If, instead, for the second central charge, we had a different phase angle θ', then this would not be the case and spin reduction would not work. With (3.28) we get

$$\delta\lambda_k = (\not{\partial} - i(\cos\theta - \sin\theta\,\gamma_5)(\alpha^1\partial_1 + \alpha^2\partial_2))(\not{A} + i\alpha^{i'}B_{i'} + i\gamma^5\beta^{j'}B'_{j'})\varepsilon_k. \tag{3.29}$$

We then need to apply the conditions (3.5) to further ensure a finite number of components.

In general we get the spin-reducing equation

$$i\not{\partial}\lambda_k = (\cos\theta + \gamma_5\sin\theta)(\alpha^i\partial_i + \gamma_5\beta^j\partial'_j)\lambda_k \tag{3.30}$$

with

$$\delta\lambda_k = (\not{\partial} - i(\cos\theta - \sin\theta\,\gamma_5)(\alpha^i\partial_i - \gamma_5\beta^j\partial'_j))(\not{A} + i\alpha^{i'}B_{i'} + i\gamma^5\beta^{j'}B'_{j'})\varepsilon_k. \tag{3.31}$$

We then use (3.19) and (3.20) to limit the number of components.

4. Superfield formulation

When there are central charges present, the covariant derivatives D^i_α, $\bar{D}_{\dot{\alpha}j}$ satisfy the anticommutation relations

$$\{D^i_\alpha, D^j_\beta\} = 2\eta_{\alpha\beta}Z^{ij} \qquad \{D^i_\alpha, D_{\dot{\alpha}j}\} = 2(\not{p}\eta)_{\alpha\dot{\alpha}}\delta^i_j. \tag{4.1}$$

On our superfield we can expect to be able to take

$$Z^*_{ij} = e^{-2i\theta}Z^{ij} \tag{4.2}$$

and we can define Z^{ij} as

$$Z^{ij}=\mathrm{e}^{\mathrm{i}\theta}(\boldsymbol{\alpha}\cdot\boldsymbol{\partial}+\mathrm{i}\boldsymbol{\beta}\cdot\boldsymbol{\partial}')^{ij}=\mathrm{e}^{\mathrm{i}\theta}(\boldsymbol{\alpha}\cdot\boldsymbol{\partial}-\mathrm{i}\boldsymbol{\beta}\cdot\boldsymbol{\partial}')_{ij} \tag{4.3}$$

where we take $\boldsymbol{\alpha}^{ij}=\boldsymbol{\alpha}_{ij}$, $\boldsymbol{\beta}^{ij}=-\boldsymbol{\beta}_{ij}$. This gives us

$$Z^{ij}Z^{jk*}=-(\partial^2_{(i)}+\partial'^2_{(j)})\delta^i_k. \tag{4.4}$$

From our knowledge of super-tableau calculus for one central charge (Rands and Taylor 1983) we can expect as our superfield

$$\phi_{j_1j_2}\sim\begin{array}{|c|}\hline \\ \hline \\ \hline\end{array}$$

with

$$Z^{j_1j_2}\phi_{j_1j_2}=0 \tag{4.5}$$

$$\phi^*_{j_1j_2}=\phi^{j_1j_2}=\tfrac{1}{2}\varepsilon^{j_1j_2j_3j_4}\phi_{j_3j_4} \tag{4.6}$$

(we will use ε to raise and lower skew symmetric indices). This transforms as

$$\boxed{\cdot}\otimes\begin{array}{|c|}\hline \\ \hline \\ \hline\end{array}=\begin{array}{|c|}\hline \\ \hline \\ \hline \cdot\\ \hline\end{array}$$

$$D^i_\alpha\phi_{j_1j_2}=-\tfrac{2}{5}D^m_\alpha\phi_{m[j_1}\delta^i_{j_2]T} \tag{4.7}$$

where this is traceless with respect to $Z^{j_1j_2}$.

We have, as independent components

$$\begin{array}{ccc}
\begin{array}{|c|}\hline \\ \hline \\ \hline\end{array} & \begin{array}{|c|}\hline \\ \hline \\ \hline \cdot\\ \hline\end{array} & \begin{array}{|c|}\hline \\ \hline \\ \hline \cdot\\ \hline \cdot\\ \hline\end{array} \\
\phi_{j_1j_2} \qquad & \qquad \psi_{\alpha_j}=D^m_\alpha\phi_{mj} \qquad & \qquad V_{\alpha\beta}=D^m_\alpha D^n_\beta\phi_{mn}
\end{array}$$

plus central charge derivatives.

That (4.7) gives us the spin-reducing equation (3.28) follows from the identities

$$Z_{mn}\phi^{nr}=-Z^{nr}\phi_{mn} \tag{4.8}$$

$$D^{(i_1i_2}\phi^{j_1)j_2}=0 \tag{4.9}$$

where $D^{i_1i_2}=\frac{1}{2}D^{(i_1}_\alpha D^{\alpha i_2)}$. Equation (4.9) gives us

$$D^{i_1(j_1}Z_{i_1i_2}\phi^{j_2)i_2}=0 \tag{4.10}$$

and together with (4.8) we can derive

$$D^{im}\phi_{mj}=8Z^{im}\phi_{mj}. \tag{4.11}$$

This is enough to give

$$\not{p}^\alpha_{\dot\alpha}D^m_\alpha\phi_{mi}=Z_{in}D_{\dot\alpha m}\phi^{mn} \tag{4.12}$$

which is our spin-reducing equation (3.28).

For more than one central charge conditions (3.19) and (3.20) need to be applied to the superfield but these are not needed for the single case.

That we need conditions (3.9) and (3.20) explicitly can be seen by the initial transformation laws (3.8) and (3.10), and looking for constraints involving fewer powers of $\boldsymbol{D}_\alpha$ than the four required for (3.19) and (3.20).

If we consider our superfield to be similar to any of the fields (3.9) we get the corresponding transformation law from (3.10) as the result of hitting the superfield with $\bar{\varepsilon}D(=\varepsilon^{\dot{\alpha}}D_{\dot{\alpha}}+\varepsilon^{\alpha}D_{\alpha})$. The action of two D_α's then corresponds to equation (3.8) and three then gives us (3.10) plus all single central charge derivatives of (3.10).

Hence we have, for example,

$$\delta V_\mu = 2\mathrm{i}\bar{\varepsilon}\gamma_{\mu'}\lambda \qquad \delta A_{1\mu} = \delta\partial_1 V_\mu = 2\mathrm{i}\bar{\varepsilon}\gamma_{\mu'}\partial_1\lambda \qquad \delta A_{2\mu} = \delta\partial_2 V_\mu = 2\mathrm{i}\bar{\varepsilon}\gamma_{\mu'}\partial_2\lambda. \tag{4.13}$$

If we were to define

$$\mathrm{i}\partial_1\lambda = \not{\partial}\tau \tag{4.14}$$

this gives us, using (2.8),

$$\delta A_{1\mu} = 2\bar{\varepsilon}\gamma_{\mu'}\not{\partial}\tau \qquad \delta A_{2\mu} = 2\bar{\varepsilon}\gamma_{\mu'}\not{\partial}(\alpha^2\lambda + \alpha^2\alpha^1\tau). \tag{4.15}$$

Since we get similar terms for the scalars it is clear that up to and including the D^3 level the parameters a_1, a_2 need not enter and hence D^4 conditions are needed to define them.

5. Discussion

We have constructed a representation of $N=4$ SUSY which has a finite number of components but has two off-shell central charges along different directions in the central charge space. This representation may be of value in the construction of $N=4$-SYM or $N=4$-SGR as helping to avoid the $N=3$ barrier. Indeed, if the results of an earlier analysis of $N=4$ SGR is accepted (Taylor 1982b) at least two such central charges are essential to bypass the $N=3$ barrier. We cannot immediately conclude from this, however, that the multiplet we have presented will be satisfactory. It may be that two single off-shell central charges are needed on two independent multiplets, and that a total of six such multiplets, each with a central charge in a different direction, will be required.

In order to determine the exact nature of the multiplets needed to be used as additional compensating multiplets, beyond those without central charges, we must determine suitable constraints on the torsions (for N-SGRs) or field strengths (for 4-SYM). These constraints must at least produce the spin-reducing constraint (2.7). Since this is too weak to prevent multiplets with an infinite number of components from appearing, either a further condition of the form (3.5) or conditions giving the set of different multiplets with single central charges must be used.

We would expect the corresponding geometric constraints to be those which allow the corresponding representation to be present (Gates 1979, Stelle and West 1979) in the full geometry. For the case of multiplets with single central charges this presents no difficulties, but the present multiplet might be more difficult to incorporate in such a geometric fashion. This is because the constraints on the superfield ϕ_{ij} to single out this multiplet which depend on the constants a_1, a_2 of (3.5) are fourth order in $D^i_\alpha, D_{\dot{\alpha}i}$, not of first order as for the single central charge case. It will be interesting to see if an argument can be given proving that this geometrisation of our two-central charge multiplet is or is not possible.

We conclude that our multiplet is of interest in its own right as possessing effectively two extra 'times' in a very non-trivial fashion, but that its value in bypassing the $N=3$ barrier is still uncertain.

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